Cosmological Tensor Perturbations in Brane Models

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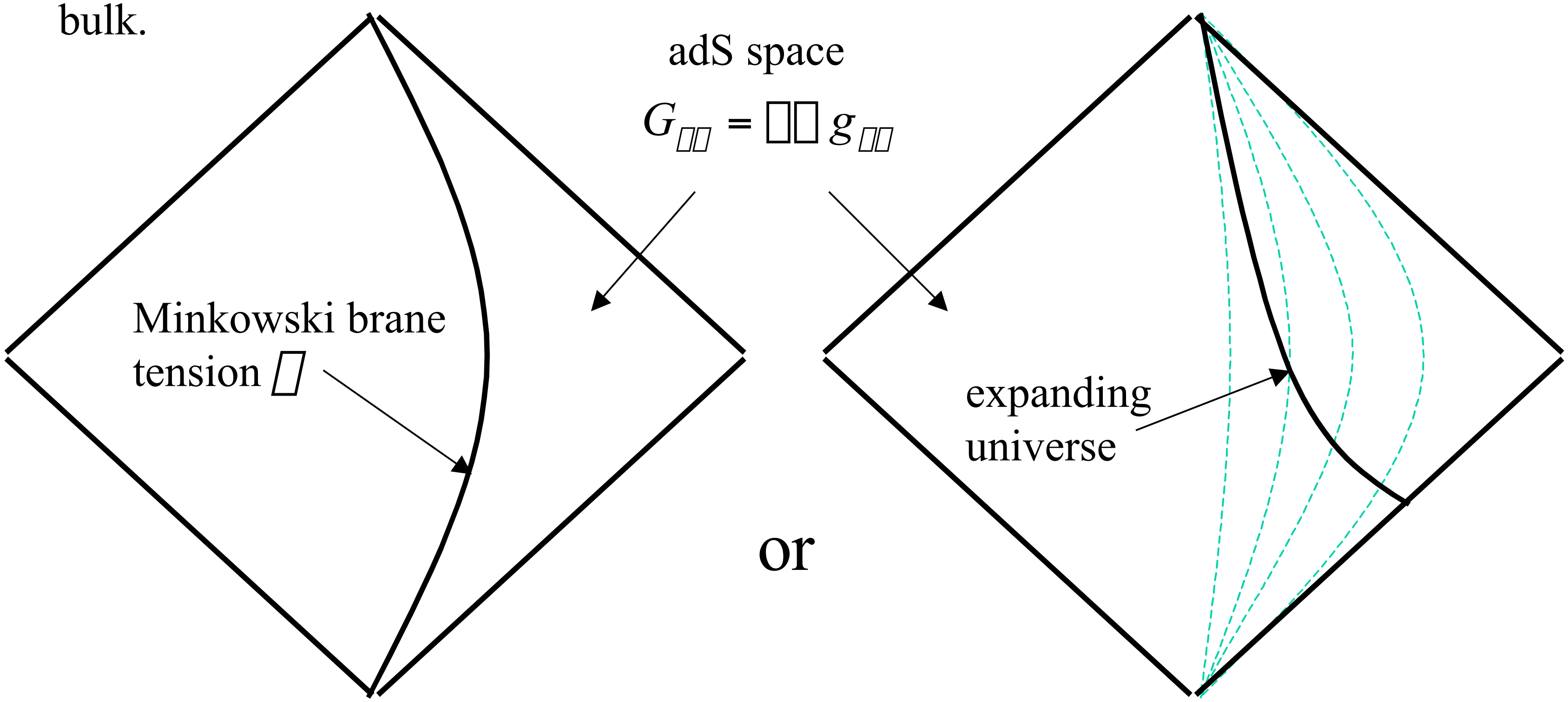
work in progress with R.A. Battye and C. van de Bruck

Randall-Sundrum model (RS2)

Our universe is a brane with large positive bare tension in an anti-de Sitter (adS) bulk.



Can calculate the following expression for the 4D Einstein tensor (Shiromizu *et al.*)

$$ {}^{(4)}G_{\mu\nu} = \tfrac{1}{2}\kappa_5\left(\Lambda + \tfrac{1}{6}\kappa_5\lambda^2\right){}^{(4)}g_{\mu\nu} + \tfrac{1}{6}\kappa_5^2\lambda T_{\mu\nu} + Q_{\mu\nu} - {}^{(4)}C^{\alpha}{}_{\mu\beta\nu}n_{\alpha}n^{\beta} $$

➢To get realistic gravity, we must tune the brane tension against the bulk cosmological constant, so as to make the first term vanish.

➢The second term is the usual r.h.s. of the Einstein equations

➢The third term represents terms quadratic in the energy-momentum.

➢In the last term, *n* is the unit vector normal to the brane and *C* is the Weyl tensor. This term represents brane-bulk gravitational interactions.

For a cosmological solution, get a modified Friedmann equation (pure adS bulk)

$$H^2 + Ka^{\square 2} = \tfrac{1}{18}\square_5^2 \square\square + \tfrac{1}{36}\square_5^2 \square^2$$

Cosmological tensor perturbations

We will use Gaussian normal coordinates, where the brane is normal to one of the axes. The flat cosmological solution has a line element of the form

$$ds^2 = n^2(\square,\square)d\square^2 + a^2(\square,\square)(\square_{ij} + h_{ij})dx^i dx^j + d\square^2$$

Can solve for *a* and *n* in the cosmological background case (Binetruy *et al*.)

$$a(\square,\square) = a(\square,0)\left[e^{\square\square/l} \square \frac{\square}{\square}\sinh(\square/l)\right] \qquad n(\square,\square) = \frac{\dot{a}(\square,\square)\, a(\square,0)}{\dot{a}(\square,0)}$$

Here we have chosen a pure adS background and conformal time and *l* is the adS lengthscale.

Linearise the Einstein equations to get equation of motion for tensor perturbations

$$\ddot{h} + \left(3\frac{\dot{a}}{a} \square \frac{\dot{n}}{n}\right)\dot{h} + k^2\frac{n^2}{a^2}h = n^2\left(3\frac{a'}{a} + \frac{n'}{n}\right)h' + n^2 h\square$$

We have Fourier transformed in the x^i directions, with k^i the transform variable.

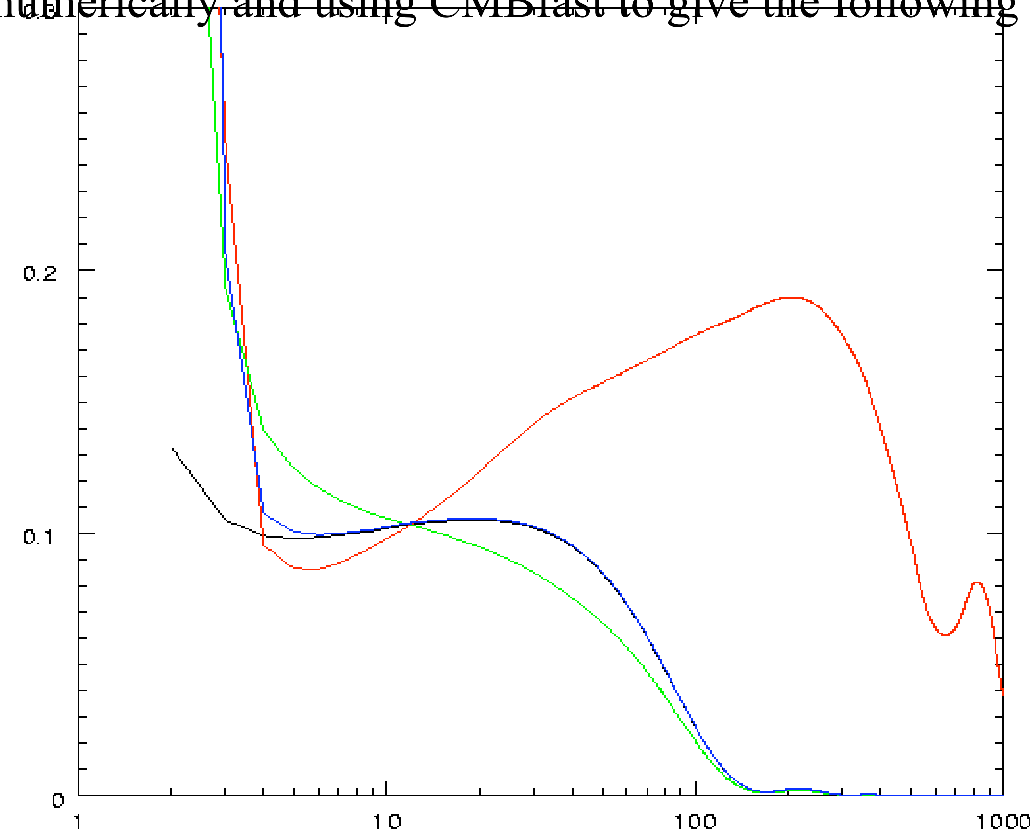
Can approximate the equation near to the brane, making it separable, writing

$$a(\square,\square) \square n(\square,\square) \square a(\square,0)e^{\square\square/l} \qquad h(\square,\square) = \square(\square)\square(\square)$$

Giving the following equation of motion for $\square$

$$\ddot{\square} + 2\frac{\dot{a}}{a}\dot{\square} + \left[k^2 + m^2 a^2\right]\square = 0$$

When $m = 0$, this looks like the standard equation for tensor perturbations in 4D cosmology. So we can gain a phenomenological understanding of tensor perturbations in brane-world models by setting m to some non-zero value, solving the equation numerically and using CMBfast to give the following power spectra.



$m = 0$ (standard 4D case)

$m = 10^{-3}$ Mpc^{-1}

$m = 1$ Mpc^{-1}

$m = 10^{3}$ Mpc^{-1}

Conclusions

- A large amount of power is produced on large angular scales. Unfortunately, this will be difficult to measure due to cosmic variance.
- For large *m* values, we see a broad peak around 200 and considerably more power on smaller scales than in the usual 4D case.
- Scalar perturbations are more difficult, but watch this space!
- We need to know the relative abundances of the various modes for a particular theory, such as RS2, in order to rule it out.
- We see that higher dimensional theories and theories with massive gravity tend to break scale invariance in the power spectrum.
- This phenomenological method has the advantage of applying to a large class of models, not just RS2.